MATHEMATICAL IDEAS

THIRTEENTH EDITION

CHARLES D. MILLER | VERN E. HEEREN | JOHN HORNSBY | CHRISTOPHER HEEREN

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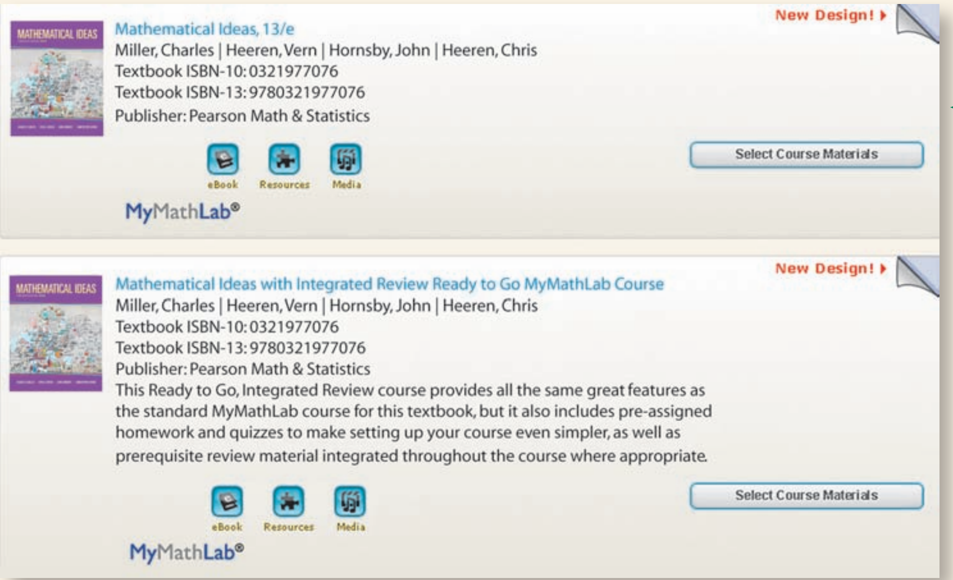

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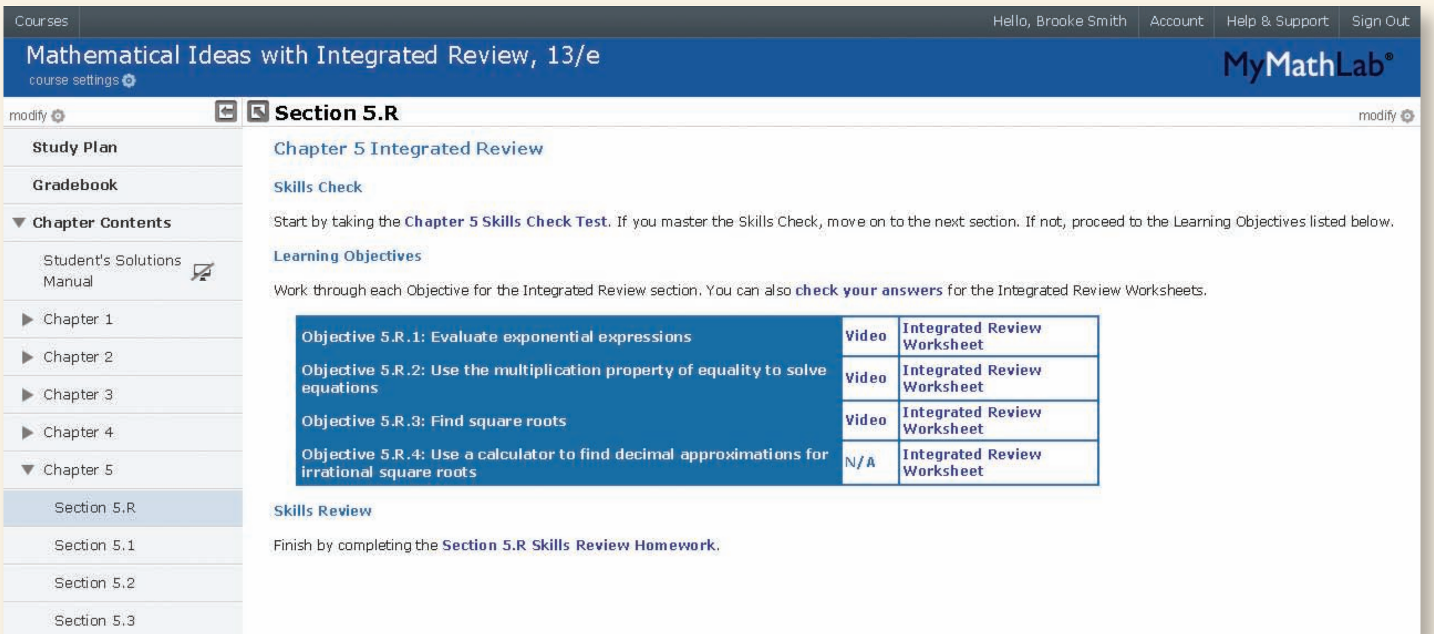
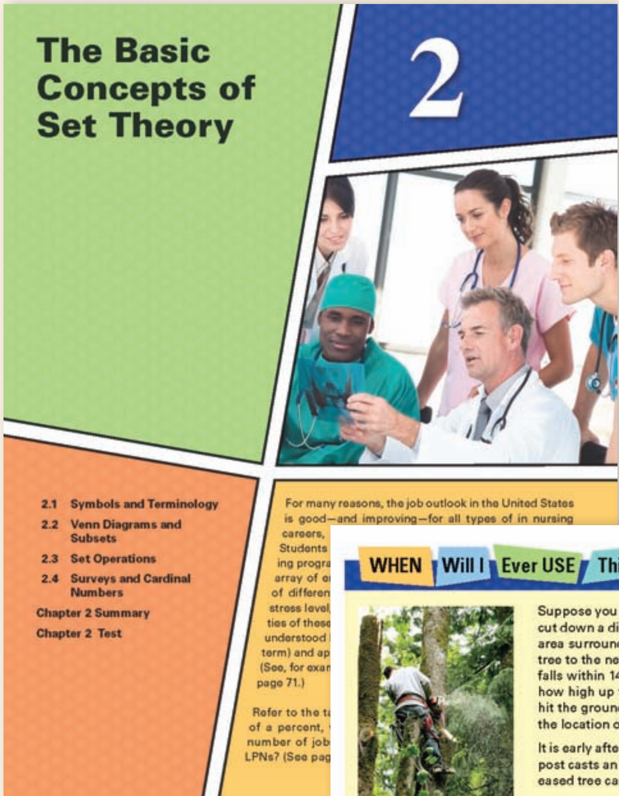
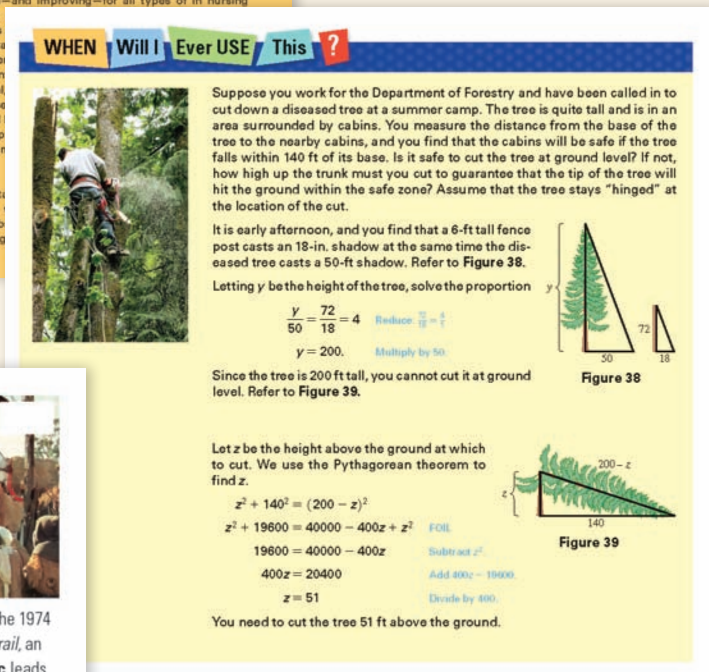


Mathematical Ideas captures the interest of non-majors by inspiring them to see mathematics as something interesting, relevant, and utterly practical. With a new focus on jobs, professions and careers as the context in which to frame the math, *Mathematical Ideas* demonstrates the importance math can play in everyday life while drawing students into the content.



All **Chapter Openers** in this edition have been updated to reflect career-based applications. This example from Chapter 2 relates the basic concepts of set theory to choosing a career path in the health care field.



WHEN Will I Ever USE This ?

Suppose you work for the Department of Forestry and have been called in to cut down a diseased tree at a summer camp. The tree is quite tall and is in an area surrounded by cabins. You measure the distance from the base of the tree to the nearby cabins, and you find that the cabins will be safe if the tree falls within 140 ft of its base. Is it safe to cut the tree at ground level? If not, how high up the trunk must you cut to guarantee that the tip of the tree will hit the ground within the safe zone? Assume that the tree stays "hinged" at the location of the cut.

It is early afternoon, and you find that a 6-ft tall fence post casts an 18-in. shadow at the same time the diseased tree casts a 50-ft shadow. Refer to **Figure 38.**

Letting y be the height of the tree, solve the proportion

$$\frac{y}{50} = \frac{72}{18} = 4 \quad \text{Reduce: } \frac{72}{18} = \frac{4}{1}$$
$$y = 200. \quad \text{Multiply by 50.}$$

Figure 38

Since the tree is 200 ft tall, you cannot cut it at ground level. Refer to **Figure 39.**

Let z be the height above the ground at which to cut. We use the Pythagorean theorem to find z.

$$z^2 + 140^2 = (200 - z)^2$$
$$z^2 + 19600 = 40000 - 400z + z^2 \quad \text{FOIL}$$
$$19600 = 40000 - 400z \quad \text{Subtract } z^2.$$
$$400z = 20400 \quad \text{Add } 400z - 19600.$$
$$z = 51 \quad \text{Divide by 400.}$$

Figure 39

You need to cut the tree 51 ft above the ground.

A brand-new feature, **When Will I Ever Use This?**, highlights how the mathematical concepts covered in the chapter might be used in a particular field. Career examples range from video game programming to nursing to forestry.

In a scene near the beginning of the 1974 film *Monty Python and the Holy Grail*, an amazing application of **poor logic** leads to the apparent demise of a supposed witch. Some peasants have forced a young woman to wear a nose made of wood. The convoluted argument they make is this: Witches and wood are both burned, and because witches are made of wood, and wood floats, and ducks also float, if she weighs the same as a duck, then she is made of wood and, therefore, is a witch!

Margin notes appear frequently throughout the text. These provide relevant examples from media and literature in addition to historical anecdotes and current research.

THIRTEENTH EDITION

MATHEMATICAL IDEAS

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Phor my phriend Phlash Phelps, whose Phunny Pharm helps me get through my mornings—JOHNNY (1119)

To my beloved wife, Carole, for decades of inspiration and support—VERN

To Heather, for your undying love and encouragement—CHRIS

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NOTE: Trigonometry module and Metrics module available in MyMathLab or online at www.pearsonhighered.com/mathstatsresources.

PREFACE

After twelve editions and over four decades, *Mathematical Ideas* continues to be one of the most popular textbooks in liberal arts mathematics education. We are proud to present the thirteenth edition of a text that offers non-physical science students a practical coverage that connects mathematics to the world around them. It is a flexible text that has evolved alongside changing trends but remains steadfast to its original objectives.

Mathematical Ideas is written with a variety of students in mind. It is well suited for several courses, including those geared toward the aforementioned liberal arts audience and survey courses in mathematics or finite mathematics. Students taking these courses will pursue careers in nursing and healthcare, the construction trades, communications, hospitality, information technology, criminal justice, retail management and sales, computer programming, political science, school administration, and a myriad of other careers. Accordingly, we have chosen to increase our focus on showcasing how the math in this course will be relevant in this wide array of career options.

- Chapter openers now address how the chapter topics can be applied within the context of work and future careers.
- We made sure to retain the hundreds of examples and exercises from the previous edition that pertain to these interests.
- Every chapter also contains the brand new ***When Will I Ever Use This?*** features that help students connect mathematics to the workplace.

Interesting and mathematically pertinent movie and television applications and references are still interspersed throughout the chapters.

Ample topics are included for a two-term course, yet the variety of topics and flexibility of sequence makes the text suitable for shorter courses as well. Our main objectives continue to be comprehensive coverage, appropriate organization, clear exposition, an abundance of examples, and well-planned exercise sets with numerous applications.

New to This Edition

- New chapter openers connect the mathematics of the chapter to a particular career area, or in some cases, to an everyday life situation that will be important to people in virtually any career.
- *When Will I Ever Use This*? features in each chapter also connect chapter topics to career or workplace situations and answer that age-old question.
- Career applications have taken on greater prominence.
- Every section of every chapter now begins with a list of clear learning objectives for the student.
- An extensive summary at the end of each chapter includes the following components.
    - A list of ***Key Terms*** for each section of the chapter
    - ***New Symbols,*** with definitions, to clarify newly introduced symbols
    - ***Test Your Word Power*** questions that allow students to test their knowledge of new vocabulary
    - A ***Quick Review*** that gives a brief summary of concepts covered in the chapter, along with examples illustrating those concepts
- All exercise sets have once again been updated, with over 1000 new or modified exercises, many with a new emphasis on career applications.

- Since Intermediate Algebra is often a prerequisite for the liberal arts course, the algebra chapters have been streamlined to focus in on key concepts, many of which will aid in comprehension of other chapters' content.
- The presentation has been made more uniform whenever clarity for the reader could be served.
- The general style has been freshened, with more pedagogical use of color, new photos and art, and opening of the exposition.
- NEW! An Integrated Review MyMathLab course option provides embedded review of select developmental topics in a Ready to Go format with assignments pre-assigned. This course solution can be used in a co-requisite course model, or simply to help under-prepared students master prerequisite skills and concepts.
- Expanded online resources
    - **NEW! Interactive, conceptual videos** with assignable MML questions walk students through a concept and then ask them to answer a question within the video. If students answer correctly, the concept is summarized. If students select one of the two incorrect answers, the video continues focusing on why students probably selected that answer and works to correct that line of thinking and explain the concept. Then students get another chance to answer a question to prove mastery.
    - **NEW! Learning Catalytics** This student engagement, assessment and classroom intelligence system gives instructors real-time feedback on student learning.
    - **NEW! "When Will I Ever Use This?" videos** bring the ideas in the feature to life in a fun, memorable way.
    - **NEW!** An **Integrated Review MyMathLab** course option provides an embedded review of selected developmental topics. Assignments are pre-assigned in this course, which includes a Skills Check quiz on skills that students will need in order to learn effectively at the chapter level. Students who demonstrate mastery can move on to the *Mathematical Ideas* content, while students who need additional review can polish up their skills by using the videos supplied and can benefit from the practice they gain from the Integrated Review Worksheets. This course solution can be used either in a co-requisite course model, or simply to help underprepared students master prerequisite skills and concepts.
    - The Trigonometry and Metrics content that was previously in the text is now found in the MyMathLab course, including the assignable MML questions.
    - Extensions previously in the text are now found in the MyMathLab course, along with any assignable MML questions.

Overview of Chapters

- **Chapter 1 (The Art of Problem Solving)** introduces the student to inductive reasoning, pattern recognition, and problem-solving techniques. We continue to provide exercises based on the monthly Calendar from *Mathematics Teacher* and have added new ones throughout this edition. The new chapter opener recounts the solving of the Rubik's cube by a college professor. The *When Will I Ever Use This?* feature (p. 31) shows how estimation techniques may be used by a group home employee charged with holiday grocery shopping.
- **Chapter 2 (The Basic Concepts of Set Theory)** includes updated examples and exercises on surveys. The chapter opener and the *When Will I Ever Use This?* feature (p. 73) address the future job outlook for the nursing profession and the allocation of work crews in the building trade, respectively.

- **Chapter 3 (Introduction to Logic)** introduces the fundamental concepts of inductive and deductive logic. The chapter opener connects logic with fantasy literature, and new exercises further illustrate this relationship. A new *For Further Thought* (p. 99) and new exercises address logic gates in computers. One *When Will I Ever Use This?* feature (p. 108) connects circuit logic to the design and installation of home monitoring systems. Another (p. 128) shows a pediatric nurse applying a logical flowchart and truth tables to a child's vaccination protocol.
- **Chapter 4 (Numeration Systems)** covers historical numeration systems, including Egyptian, Roman, Chinese, Babylonian, Mayan, Greek, and Hindu-Arabic systems. A connection between base conversions in positional numeration systems and computer network design is suggested in the new chapter opener and illustrated in the *When Will I Ever Use This?* feature (p. 168), a new example, and new exercises.
- **Chapter 5 (Number Theory)** presents an introduction to the prime and composite numbers, the Fibonacci sequence, and a cross section of related historical developments, including the fairly new topic of "prime number splicing." The largest currently known prime numbers of various categories are identified, and recent progress on Goldbach's conjecture and the twin prime conjecture are noted. The chapter opener and one *When Will I Ever Use This?* feature (p. 189) apply cryptography and modular arithmetic to criminal justice, relating to cyber security. Another *When Will I Ever Use This*? feature (p. 205) shows how a nurse may use the concept of least common denominator in determining proper drug dosage.
- **Chapter 6 (The Real Numbers and Their Representations)** introduces some of the basic concepts of real numbers, their various forms of representation, and operations of arithmetic with them. The chapter opener and *When Will I Ever Use This?* feature (p. 273) connect percents and basic algebraic procedures to pricing, markup and discount, student grading, and market share analysis, as needed by a retail manager, a teacher, a salesperson, a fashion merchandiser, and a business owner.
- **Chapter 7 (The Basic Concepts of Algebra)** can be used to present the basics of algebra (linear and quadratic equations, applications, exponents, polynomials, and factoring) to students for the first time, or as a review of previous courses. The chapter opener connects proportions to an automobile owner's determination of fuel mileage, and the *When Will I Ever Use This?* feature (p. 330) relates inequalities to a test-taker's computation of the score needed to maintain a certain grade point average.
- **Chapter 8 (Graphs, Functions, and Systems of Equations and Inequalities)** is the second of our two algebra chapters. It continues with graphs, equations, and applications of linear, quadratic, exponential, and logarithmic functions and models, along with systems of equations. The chapter opener shows how an automobile owner can use a linear graph to relate price per gallon, amount purchased, and total cost. The *When Will I Ever Use This?* feature (p. 416) connects logarithms with the interpretation of earthquake reporting in the news.
- **Chapter 9 (Geometry)** covers elementary plane geometry, transformational geometry, basic geometric constructions, non-Euclidean geometry, and chaos and fractals. Section 9.7 now includes projective geometry. At reviewer request, the discussion of networks (the Königsberg Bridge problem) has been moved to Chapter 14 (Graph Theory). The chapter opener and one *When Will I Ever Use This?* feature (p. 497) connect geometric volume formulas to a video game programmer's job of designing the visual field of a game screen. A second *When Will I Ever Use This?* feature (p. 470) relates right triangle geometry to a forester's determining of safe tree-felling parameters.

- **Chapter 10 (Counting Methods)** focuses on elementary counting techniques, in preparation for the probability chapter. The chapter opener relates how a restaurateur used counting methods to help design the sales counter signage in a new restaurant. The *When Will I Ever Use This?* feature (p. 534) describes an entrepreneur's use of probability and sports statistics in designing a game and in building a successful company based on it.

- **Chapter 11 (Probability)** covers the basics of probability, odds, and expected value. The chapter opener relates to the professions of weather forecaster, actuary, baseball manager, and corporate manager, applying probability, statistics, and expected value to interpreting forecasts, determining insurance rates, selecting optimum strategies, and making business decisions. One *When Will I Ever Use This?* feature (p. 586) shows how a tree diagram helps a decision maker provide equal chances of winning to three players in a game of chance. A second such feature (p. 606) shows how knowledge of probability can help a television game show contestant determine the best winning strategy.

- **Chapter 12 (Statistics)** is an introduction to statistics that focuses on the measures of central tendency, dispersion, and position and discusses the normal distribution and its applications. The chapter opener and two *When Will I Ever Use This?* features (pp. 656, 661) connect probability and graph construction and interpretation to how a psychological therapist may motivate and carry out treatment for alcohol and tobacco addiction.

- **Chapter 13 (Personal Financial Management)** provides the student with the basics of the mathematics of finance as applied to inflation, consumer debt, and house buying. We also include a section on investing, with emphasis on stocks, bonds, and mutual funds. Tables, examples, and exercises have been updated to reflect current interest rates and investment returns. New margin notes feature smart apps for financial calculations. Additions in response to reviewer requests include a *When Will I Ever Use This?* feature (p. 741) connecting several topics of the chapter to how a financial planner can provide comparisons between renting and buying a house, and exercises comparing different mortgage options. Another *When Will I Ever Use This?* feature (p. 732) explores the cost-effectiveness of solar energy, using chapter topics essential for a solar energy salesperson. The chapter opener connects the time value of money to how a financial planner can help clients make wise financial decisions.

- **Chapter 14 (Graph Theory)** covers the basic concepts of graph theory and its applications. The chapter opener shows how a writer can apply graph theory to the analysis of poetic rhyme. One *When Will I Ever Use This?* feature (p. 800) connects graph theory to how a postal or delivery service manager could determine the most efficient delivery routes. Another (p. 818) tells of a unique use by an entrepreneur who developed a business based on finding time-efficient ways to navigate theme parks.

- **Chapter 15 (Voting and Apportionment)** deals with issues in voting methods and apportionment of representation, topics that have become increasingly popular in liberal arts mathematics courses. The Adams method of apportionment, as well as the Huntington-Hill method (currently used in United States presidential elections) are now included in the main body of the text. To illustrate the important work of a political consultant, the chapter opener connects different methods of analyzing votes. One *When Will I Ever Use This?* feature (p. 859) relates voting methods to the functioning of governing boards. Another (p. 874) gives an example of how understanding apportionment methods can help in the work of a school administrator.

Course Outline Considerations

Chapters in the text are, in most cases, independent and may be covered in the order chosen by the instructor. The few exceptions are as follows:

- Chapter 6 contains some material dependent on the ideas found in Chapter 5.
- Chapter 6 should be covered before Chapter 7 if student background so dictates.
- Chapters 7 and 8 form an algebraic "package" and should be covered in sequential order.
- A thorough coverage of Chapter 11 depends on knowledge of Chapter 10 material, although probability can be covered without teaching extensive counting methods by avoiding the more difficult exercises.

Features of the Thirteenth Edition

NEW! Chapter Openers In keeping with the career theme, chapter openers address a situation related to a particular career. All are new to this edition. Some openers include a problem that the reader is asked to solve. We hope that you find these chapter openers useful and practical.

ENHANCED! Varied Exercise Sets We continue to present a variety of exercises that integrate drill, conceptual, and applied problems, and there are over 1000 new or modified exercises in this edition. The text contains a wealth of exercises to provide students with opportunities to practice, apply, connect, and extend the mathematical skills they are learning. We have updated the exercises that focus on real-life data and have retained their titles for easy identification. Several chapters are enriched with new applications, particularly Chapters 6, 7, 8, 11, 12, and 13. We continue to use graphs, tables, and charts when appropriate. Many of the graphs use a style similar to that seen by students in today's print and electronic media.

UPDATED! Emphasis on Real Data in the Form of Graphs, Charts, and Tables We continue to use up-to-date information from magazines, newspapers, and the Internet to create real applications that are relevant and meaningful.

Problem-Solving Strategies Special paragraphs labeled "Problem-Solving Strategy" relate the discussion of problem-solving strategies to techniques that have been presented earlier.

For Further Thought These entries encourage students to share their reasoning processes among themselves to gain a deeper understanding of key mathematical concepts.

ENHANCED! Margin Notes This popular feature is a hallmark of this text and has been retained and updated where appropriate. These notes are interspersed throughout the text and are drawn from various sources, such as lives of mathematicians, historical vignettes, anecdotes on mathematics textbooks of the past, newspaper and magazine articles, and current research in mathematics.

Optional Graphing Technology We continue to provide sample graphing calculator screens to show how technology can be used to support results found analytically. It is not essential, however, that a student have a graphing calculator to study from this text. *The technology component is optional.*

NEW! Chapter Summaries Extensive summaries at the end of each chapter include Key Terms, New Symbols with definitions, Test Your Word Power vocabulary checks, and a Quick Review that provides a brief summary of concepts (with examples) covered in the chapter.

Chapter Tests Each chapter concludes with a chapter test so that students can check their mastery of the material.

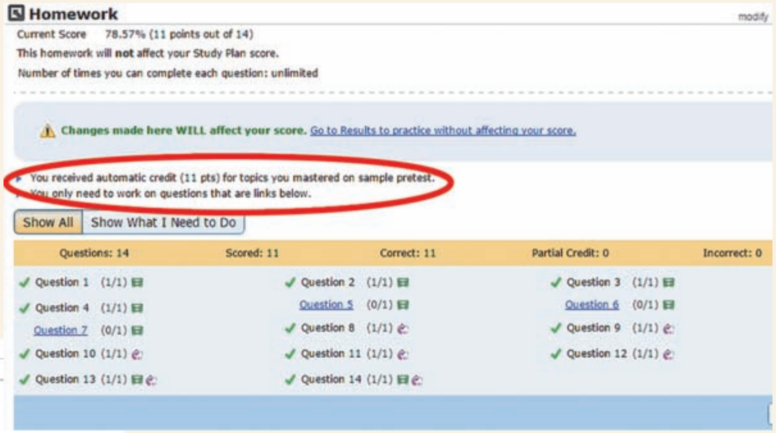
Resources For Success

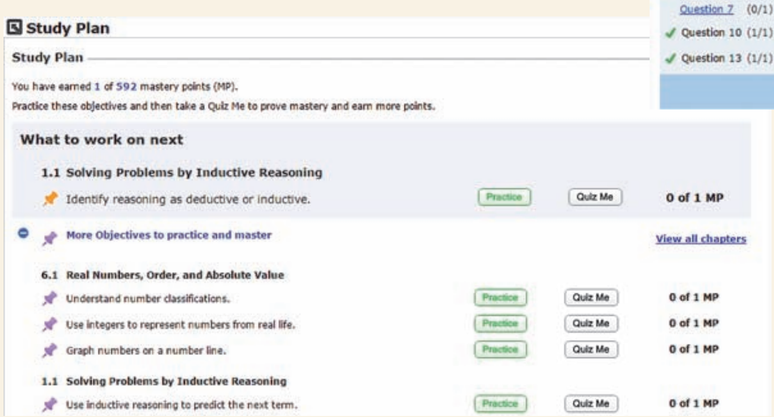
MyMathLab® Online Course (access code required)

MyMathLab delivers **proven results** in helping individual students succeed. It provides **engaging experiences** that personalize, stimulate, and measure learning for each student. And it comes from an **experienced partner** with educational expertise and an eye on the future. **MyMathLab** helps prepare students and gets them thinking more conceptually and visually through the following features:

Personalized Homework ▸

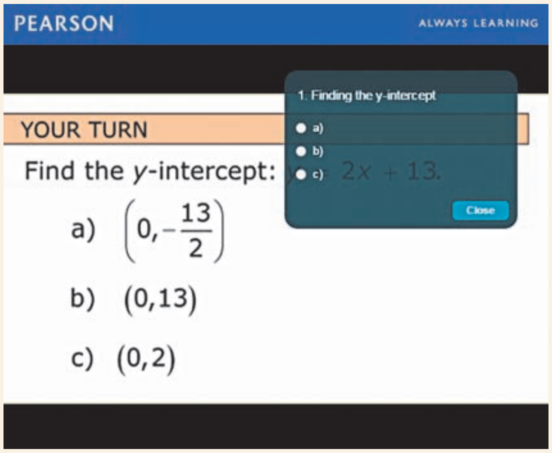
Attaching a personalized homework assignment to a quiz or test enables students to focus on the topics they did not master and those with which they need additional practice, providing an individualized experience.





◂ Adaptive Study Plan

The Study Plan makes studying more efficient and effective for every student. Performance and activity are assessed continually in real time. Then data and analytics are used to provide personalized content-reinforcing concepts that target each student's strengths and weaknesses.

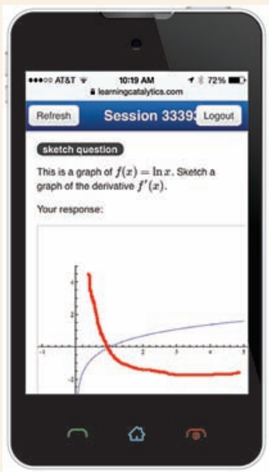


◂ Interactive Concept Videos

NEW! Conceptual videos require students' input as they walk students through a concept and pause to ask them questions. Correct answers are followed by reinforcement of the concept. Incorrect answers are followed by a video that explains the correct answer, as well as addressing the misconception that may have led to that particular mistake. Instructors can also assign trackable exercises that correspond to the videos to check student understanding.

Learning Catalytics ▸

NEW! Integrated into **MyMathLab**, the Learning Catalytics feature uses students' devices in the classroom for an engagement, assessment, and classroom intelligence system that gives instructors real-time feedback on student learning.



Skills for Success Modules are integrated within the **MyMathLab** course to help students succeed in college courses and prepare for future professions.

Instructor Resources

Additional resources can be downloaded from www.pearsonhighered.com or hardcopy resources can be ordered from your sales representative.

Integrated Review Ready to Go MyMathLab® Course

The Ready to Go **MyMathLab** course option makes it even easier to get started, including author-chosen preassigned homework, integrated review of prerequisite topics, and more.

TestGen®

TestGen® (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text.

PowerPoint® Lecture Slides

Fully editable slides correlated with the textbook are available.

Annotated Instructor's Edition

When possible, answers are on the page with the exercises. Longer answers are in the back of the book.

Instructor's Resource and Solutions Manual

This manual includes fully worked solutions to all text exercises, as well as the Collaborative Investigations that were formerly in the text.

Instructor's Testing Manual

This manual includes tests with answer keys for each chapter of the text.

Student Resources

Additional resources are available to support student success.

Updated Video Program

Available in **MyMathLab**, video lectures cover every section in the text and have been updated for this edition where necessary. New interactive concept videos and new "*When Will I Ever Use This?*" videos complete the video package, reinforcing students' conceptual understanding, while also engaging them with the math in context.

Student Solutions Manual

This manual provides detailed worked-out solutions to odd-numbered exercises.

Integrated Review Worksheets

Intended to be used with the Integrated Review **MyMathLab** course, these worksheets give students an opportunity to review and practice prerequisite topics from developmental math that are needed for each chapter in *Mathematical Ideas*.

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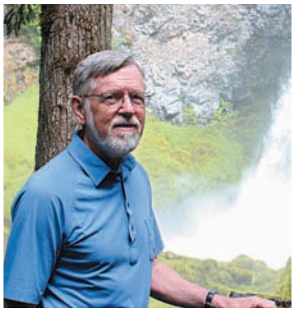
Vern E. Heeren
John Hornsby
Christopher Heeren

ABOUT THE AUTHORS



Vern Heeren grew up in the Sacramento Valley of California. After earning a Bachelor of Arts degree in mathematics, with a minor in physics, at Occidental College, and completing his Master of Arts degree in mathematics at the University of California, Davis, he began a 38-year teaching career at American River College, teaching math and a little physics. He coauthored *Mathematical Ideas* in 1968 with office mate Charles Miller, and he has enjoyed researching and revising it over the years. It has been a joy for him to complete the thirteenth edition, along with long-time coauthor John Hornsby, and now also with son Christopher.

These days, besides pursuing his mathematical interests, Vern enjoys spending time with his wife Carole and their family, exploring the wonders of nature near their home in central Oregon.

John Hornsby joined the author team of Margaret Lial, Charles Miller, and Vern Heeren in 1988. In 1990, the sixth edition of *Mathematical Ideas* became the first of nearly 150 titles he has coauthored for Scott Foresman, HarperCollins, Addison-Wesley, and Pearson in the years that have followed. His books cover the areas of developmental and college algebra, precalculus, trigonometry, and mathematics for the liberal arts. He is a native and resident of New Roads, Louisiana.

Christopher Heeren is a native of Sacramento, California. While studying engineering in college, he had an opportunity to teach a math class at a local high school, and this sparked both a passion for teaching and a change of major. He received a Bachelor of Arts degree and a Master of Arts degree, both in mathematics, from California State University, Sacramento. Chris has taught mathematics at the middle school, high school, and college levels, and he currently teaches at American River College in Sacramento. He has a continuing interest in using technology to bring mathematics to life. When not writing, teaching, or preparing to teach, Chris enjoys spending time with his lovely wife Heather and their three children (and two dogs and a guinea pig).

The Art of Problem Solving

1

1.1 Solving Problems by Inductive Reasoning

1.2 An Application of Inductive Reasoning: Number Patterns

1.3 Strategies for Problem Solving

1.4 Numeracy in Today's World

Chapter 1 Summary

Chapter 1 Test

Professor Terry Krieger, of Rochester (Minnesota) Community College, shares his thoughts about why he decided to become a mathematics teacher. He is an expert at the Rubik's Cube. Here, he explains how he mastered this classic problem.

From a very young age I always enjoyed solving problems, especially problems involving numbers and patterns. There is something inherently beautiful in the process of discovering mathematical truth. Mathematics may be the only discipline in which different people, using wildly varied but logically sound methods, will arrive at the same correct result—not just once, but every time! It is this aspect of mathematics that led me to my career as an educator. As a mathematics instructor, I get to be part of, and sometimes guide, the discovery process.

I received a Rubik's Cube as a gift my junior year of high school. I was fascinated by it. I devoted the better part of three months to solving it for the first time, sometimes working 3 or 4 hours per day on it.

There was a lot of trial and error involved. I devised a process that allowed me to move only a small number of pieces at a time while keeping other pieces in their places. Most of my moves affect only three or four of the 26 unique pieces of the puzzle. What sets my solution apart from those found in many books is that I hold the cube in a consistent position and work from the top to the bottom. Most book solutions work upward from the bottom.

My first breakthrough came when I realized that getting a single color on one face of the cube was not helpful if the colors along the edges of that face were placed improperly. In other words, it does no good to make the top of the cube all white if one of the edges along the white top shows green, yellow, and blue. It needs to be all green, for example.

I worked on the solution so much that I started seeing cube moves in my sleep. In fact, I figured out the moves for one of my most frustrating sticking points while sleeping. I just woke up knowing how to do it.

The eight corners of the cube represented a particularly difficult challenge for me. Finding a consistent method for placing the corners appropriately took many, many hours. To this day, the amount of time that it takes for me to solve a scrambled cube depends largely on the amount of time that it takes for me to place the corners.

When I first honed my technique, I was able to consistently solve the cube in 2 to 3 minutes. My average time is now about 65 seconds. My fastest time is 42 seconds.

Since figuring out how to solve the cube, I have experimented with other possible color patterns that can be formed. The most complicated one I have created leaves the cube with three different color stripes on all six faces. I have never met another person who can accomplish this arrangement.

1.1 SOLVING PROBLEMS BY INDUCTIVE REASONING

OBJECTIVES

1. Be able to distinguish between inductive and deductive reasoning.
2. Understand that in some cases, inductive reasoning may not lead to valid conclusions.

Characteristics of Inductive and Deductive Reasoning

The development of mathematics can be traced to the Egyptian and Babylonian cultures (3000 B.C.–A.D. 260) as a necessity for counting and problem solving. To solve a problem, a cookbook-like recipe was given, and it was followed repeatedly to solve similar problems. By observing that a specific method worked for a certain type of problem, the Babylonians and the Egyptians concluded that the same method would work for any similar type of problem. Such a conclusion is called a *conjecture*. A **conjecture** is an educated guess based on repeated observations of a particular process or pattern.

The method of reasoning just described is called *inductive reasoning*.

INDUCTIVE REASONING

Inductive reasoning is characterized by drawing a general conclusion (making a conjecture) from repeated observations of specific examples. The conjecture may or may not be true.

In testing a conjecture obtained by inductive reasoning, it takes only one example that does not work to prove the conjecture false. Such an example is called a **counterexample.**

Inductive reasoning provides a powerful method of drawing conclusions, but there is no assurance that the observed conjecture will always be true. For this reason, mathematicians are reluctant to accept a conjecture as an absolute truth until it is formally proved using methods of *deductive reasoning.* Deductive reasoning characterized the development and approach of Greek mathematics, as seen in the works of Euclid, Pythagoras, Archimedes, and others. During the classical Greek period (600 B.C.–A.D. 450), general concepts were applied to specific problems, resulting in a structured, logical development of mathematics.

DEDUCTIVE REASONING

Deductive reasoning is characterized by applying general principles to specific examples.

We now look at examples of these two types of reasoning. In this chapter, we often refer to the **natural,** or **counting, numbers.**

1, 2, 3, . . .   Natural (counting) numbers

↑

Ellipsis points

The three dots (*ellipsis points*) indicate that the numbers continue indefinitely in the pattern that has been established. The most probable rule for continuing this pattern is "Add 1 to the previous number," and this is indeed the rule that we follow.

Now consider the following list of natural numbers:

$$2, 9, 16, 23, 30.$$

What is the next number of this list? What is the pattern? After studying the numbers, we might see that $2 + \mathbf{7} = 9$, and $9 + \mathbf{7} = 16$. Do we add 16 and 7 to get 23? Do we add 23 and 7 to get 30? Yes. It seems that any number in the given list can be found by adding 7 to the preceding number, so the next number in the list would be $30 + 7 = 37$.

We set out to find the "next number" by reasoning from observation of the numbers in the list. We may have jumped from these observations to the general statement that any number in the list is 7 more than the preceding number. This is an example of inductive reasoning.

By using inductive reasoning, we concluded that 37 was the next number. Suppose the person making up the list has another answer in mind. The list of numbers

$$2, 9, 16, 23, 30$$

actually gives the dates of Mondays in June if June 1 falls on a Sunday. The next Monday after June 30 is July 7. With this pattern, the list continues as

$$2, 9, 16, 23, 30, 7, 14, 21, 28, \ldots .$$

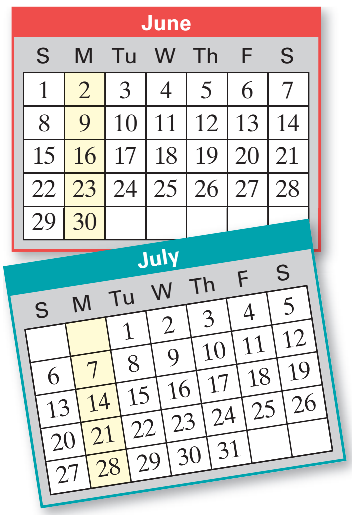


Figure 1

See the calendar in **Figure 1.** The correct answer would then be 7. The process used to obtain the rule "add 7" in the preceding list reveals a main flaw of inductive reasoning. ***We can never be sure that what is true in a specific case will be true in general. Inductive reasoning does not guarantee a true result, but it does provide a means of making a conjecture.***

We now review some basic notation. Throughout this book, we use *exponents* to represent repeated multiplication.

$$\text{Base} \rightarrow 4^3 = 4 \cdot 4 \cdot 4 = 64 \quad \text{4 is used as a factor 3 times.}$$

$\uparrow$
Exponent

EXPONENTIAL EXPRESSION

If a is a number and n is a counting number $(1, 2, 3, \ldots)$, then the exponential expression a^n is defined as follows.

$$a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ factors of } a}$$

The number a is the **base** and n is the **exponent.**

With deductive reasoning, we use general statements and apply them to specific situations. For example, a basic rule for converting feet to inches is to multiply the number of feet by 12 in order to obtain the equivalent number of inches. This can be expressed as a formula.

$$\text{Number of inches} = 12 \times \text{number of feet}$$

This general rule can be applied to any specific case. For example, the number of inches in 3 feet is $12 \times 3 = 36$ inches.

Reasoning through a problem usually requires certain *premises*. A **premise** can be an assumption, law, rule, widely held idea, or observation. Then reason inductively or deductively from the premises to obtain a **conclusion.** The premises and conclusion make up a **logical argument.**

EXAMPLE 1 Identifying Premises and Conclusions

Identify each premise and the conclusion in each of the following arguments. Then tell whether each argument is an example of inductive or deductive reasoning.

(a) Our house is made of brick. Both of my next-door neighbors have brick houses. Therefore, all houses in our neighborhood are made of brick.

(b) All keyboards have the symbol @. I have a keyboard. My keyboard has the symbol @.

(c) Today is Tuesday. Tomorrow will be Wednesday.

Solution

(a) The premises are "Our house is made of brick" and "Both of my next-door neighbors have brick houses." The conclusion is "Therefore, all houses in our neighborhood are made of brick." Because the reasoning goes from specific examples to a general statement, the argument is an example of inductive reasoning (although it may very well be faulty).

(b) Here the premises are "All keyboards have the symbol @" and "I have a keyboard." The conclusion is "My keyboard has the symbol @." This reasoning goes from general to specific, so deductive reasoning was used.

(c) There is only one premise here, "Today is Tuesday." The conclusion is "Tomorrow will be Wednesday." The fact that Wednesday immediately follows Tuesday is being used, even though this fact is not explicitly stated. Because the conclusion comes from general facts that apply to this special case, deductive reasoning was used. ■

While inductive reasoning may, at times, lead to false conclusions, in many cases it does provide correct results if we look for the most *probable* answer.

The Fibonacci Sequence

1, 1, 2, 3, 5, 8, 13, 21, ...

In the 2003 movie *A Wrinkle in Time*, young Charles Wallace, played by David Dorfman, is challenged to identify a particular sequence of numbers. He correctly identifies it as the **Fibonacci sequence.**

EXAMPLE 2 Predicting the Next Number in a Sequence

Use inductive reasoning to determine the *probable* next number in each list below.

(a) 5, 9, 13, 17, 21, 25, 29 **(b)** 1, 1, 2, 3, 5, 8, 13, 21 **(c)** 2, 4, 8, 16, 32

Solution

(a) Each number in the list is obtained by adding 4 to the previous number. The probable next number is $29 + 4 = 33$. (This is an example of an *arithmetic sequence*.)

(b) Beginning with the third number in the list, 2, each number is obtained by adding the two previous numbers in the list. That is,

$$1 + 1 = 2, \qquad 1 + 2 = 3, \qquad 2 + 3 = 5,$$

and so on. The probable next number in the list is $13 + 21 = 34$. (These are the first few terms of the *Fibonacci sequence*.)

(c) It appears here that to obtain each number after the first, we must double the previous number. Therefore, the probable next number is $32 \times 2 = 64$. (This is an example of a *geometric sequence*.) ■

EXAMPLE 3 Predicting the Product of Two Numbers

Consider the list of equations. Predict the next multiplication fact in the list.

$$\begin{aligned} 37 \times 3 &= 111 \\ 37 \times 6 &= 222 \\ 37 \times 9 &= 333 \\ 37 \times 12 &= 444 \end{aligned}$$

Solution

The left side of each equation has two factors, the first 37 and the second a multiple of 3, beginning with 3. Each product (answer) consists of three digits, all the same, beginning with 111 for 37×3. Thus, the next multiplication fact would be

$$37 \times 15 = 555, \quad \text{which is indeed true.}$$ ■

Pitfalls of Inductive Reasoning

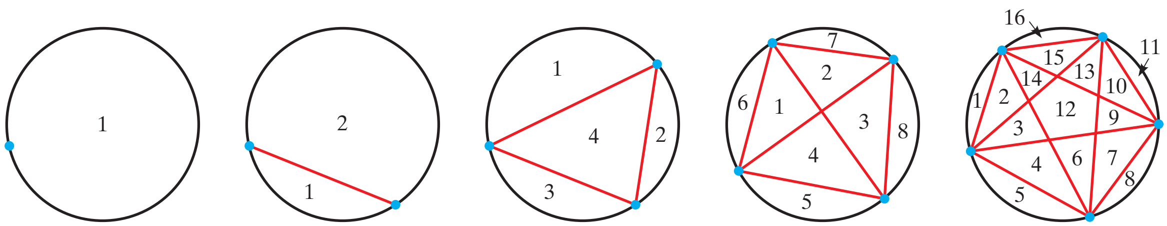
There are pitfalls associated with inductive reasoning. A classic example involves the maximum number of regions formed when chords are constructed in a circle. When two points on a circle are joined with a line segment, a *chord* is formed.

Locate a single point on a circle. Because no chords are formed, a single interior region is formed. See **Figure 2(a)** on the next page. Locate two points and draw a chord. Two interior regions are formed, as shown in **Figure 2(b).** Continue this pattern. Locate three points, and draw all possible chords. Four interior regions are formed, as shown in **Figure 2(c).** Four points yield 8 regions and five points yield 16 regions. See **Figures 2(d) and 2(e).**

The results of the preceding observations are summarized in **Table 1.** The pattern formed in the column headed "Number of Regions" is the same one we saw in **Example 2(c),** where we predicted that the next number would be 64. It seems here that for each additional point on the circle, the number of regions doubles.

Table 1

Number of Points	Number of Regions
1	1
2	2
3	4
4	8
5	16



(a) (b) (c) (d) (e)

Figure 2

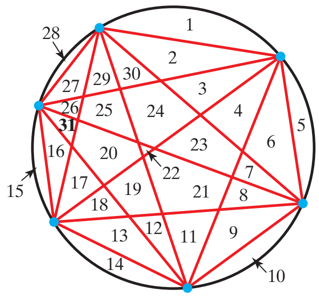


Figure 3

A reasonable inductive conjecture would be that for six points, 32 regions would be formed. But as **Figure 3** indicates, there are *only 31 regions*. The pattern of doubling ends when the sixth point is considered. Adding a seventh point would yield 57 regions. The numbers obtained here are

$$1, 2, 4, 8, 16, 31, 57.$$

For n points on the circle, the number of regions is given by the formula

$$\frac{n^4 - 6n^3 + 23n^2 - 18n + 24}{24}.^*$$

1.1 EXERCISES

In Exercises 1–16, determine whether the reasoning is an example of deductive or inductive reasoning.

1. The next number in the pattern 2, 4, 6, 8, 10 is 12.

2. My dog barked and woke me up at 1:02 a.m., 2:03 a.m., and 3:04 a.m. So he will bark again and wake me up at 4:05 a.m.

3. To find the perimeter P of a square with side of length s, I can use the formula $P = 4s$. So the perimeter of a square with side of length 7 inches is $4 \times 7 = 28$ inches.

4. A company charges a 10% re-stocking fee for returning an item. So when I return a radio that cost \$150, I will only get \$135 back.

5. If the mechanic says that it will take seven days to repair your SUV, then it will actually take ten days. The mechanic says, "I figure it'll take exactly one week to fix it, ma'am." Then you can expect it to be ready ten days from now.

6. If you take your medicine, you'll feel a lot better. You take your medicine. Therefore, you'll feel a lot better.

7. It has rained every day for the past six days, and it is raining today as well. So it will also rain tomorrow.

8. Carrie's first five children were boys. If she has another baby, it will be a boy.

9. The 2000 movie *Cast Away* stars Tom Hanks as the only human survivor of a plane crash, stranded on a tropical island. He approximates his distance from where the plane lost radio contact to be 400 miles (a radius), and uses the formula for the area of a circle,

$$\text{Area} = \pi\ (\text{radius})^2$$

to determine that a search party would have to cover an area of over 500,000 square miles to look for him and his "pal" Wilson.

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*For more information on this and other similar patterns, see "Counting Pizza Pieces and Other Combinatorial Problems," by Eugene Maier, in the January 1988 issue of *Mathematics Teacher,* pp. 22–26.

10. If the same number is subtracted from both sides of a true equation, the new equation is also true. I know that $9 + 18 = 27$. Therefore, $(9 + 18) - 13 = 27 - 13$.

11. If you build it, they will come. You build it. Therefore, they will come.

12. All men are mortal. Socrates is a man. Therefore, Socrates is mortal.

13. It is a fact that every student who ever attended Delgado University was accepted into graduate school. Because I am attending Delgado, I can expect to be accepted to graduate school, too.

14. For the past 126 years, a rare plant has bloomed in Columbia each summer, alternating between yellow and green flowers. Last summer, it bloomed with green flowers, so this summer it will bloom with yellow flowers.

15. In the sequence 5, 10, 15, 20, 25, . . . , the most probable next number is 30.

16. (This anecdote is adapted from a story by Howard Eves in *In Mathematical Circles*.) A scientist had a group of 100 fleas, and one by one he would tell each flea "Jump," and the flea would jump. Then with the same fleas, he yanked off their hind legs and repeated "Jump," but the fleas would not jump. He concluded that when a flea has its hind legs yanked off, it cannot hear.

17. Discuss the differences between inductive and deductive reasoning. Give an example of each.

18. Give an example of faulty inductive reasoning.

Determine the most probable next term in each of the following lists of numbers.

19. 6, 9, 12, 15, 18

20. 13, 18, 23, 28, 33

21. 3, 12, 48, 192, 768

22. 32, 16, 8, 4, 2

23. 3, 6, 9, 15, 24, 39

24. $\frac{1}{3}, \frac{3}{5}, \frac{5}{7}, \frac{7}{9}, \frac{9}{11}$

25. $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10}$

26. 1, 4, 9, 16, 25

27. 1, 8, 27, 64, 125

28. 2, 6, 12, 20, 30, 42

29. 4, 7, 12, 19, 28, 39

30. 27, 21, 16, 12, 9

31. 5, 3, 5, 5, 3, 5, 5, 5, 3, 5, 5, 5, 5, 3, 5, 5, 5, 5

32. 8, 2, 8, 2, 2, 8, 2, 2, 2, 8, 2, 2, 2, 2, 8, 2, 2, 2, 2

33. Construct a list of numbers similar to those in **Exercise 19** such that the most probable next number in the list is 60.

34. Construct a list of numbers similar to those in **Exercise 30** such that the most probable next number in the list is 8.

Use the list of equations and inductive reasoning to predict the next equation, and then verify your conjecture.

35.
$$\begin{aligned}(9 \times 9) + 7 &= 88\\(98 \times 9) + 6 &= 888\\(987 \times 9) + 5 &= 8888\\(9876 \times 9) + 4 &= 88{,}888\end{aligned}$$

36.
$$\begin{aligned}(1 \times 9) + 2 &= 11\\(12 \times 9) + 3 &= 111\\(123 \times 9) + 4 &= 1111\\(1234 \times 9) + 5 &= 11{,}111\end{aligned}$$

37.
$$\begin{aligned}3367 \times 3 &= 10{,}101\\3367 \times 6 &= 20{,}202\\3367 \times 9 &= 30{,}303\\3367 \times 12 &= 40{,}404\end{aligned}$$

38.
$$\begin{aligned}15873 \times 7 &= 111{,}111\\15873 \times 14 &= 222{,}222\\15873 \times 21 &= 333{,}333\\15873 \times 28 &= 444{,}444\end{aligned}$$

39.
$$\begin{aligned}34 \times 34 &= 1156\\334 \times 334 &= 111{,}556\\3334 \times 3334 &= 11{,}115{,}556\end{aligned}$$

40.
$$\begin{aligned}11 \times 11 &= 121\\111 \times 111 &= 12{,}321\\1111 \times 1111 &= 1{,}234{,}321\end{aligned}$$

41.
$$\begin{aligned}3 &= \frac{3(2)}{2}\\3 + 6 &= \frac{6(3)}{2}\\3 + 6 + 9 &= \frac{9(4)}{2}\\3 + 6 + 9 + 12 &= \frac{12(5)}{2}\end{aligned}$$

42.
$$\begin{aligned}2 &= 4 - 2\\2 + 4 &= 8 - 2\\2 + 4 + 8 &= 16 - 2\\2 + 4 + 8 + 16 &= 32 - 2\end{aligned}$$

43.
$$\begin{aligned}5(6) &= 6(6 - 1)\\5(6) + 5(36) &= 6(36 - 1)\\5(6) + 5(36) + 5(216) &= 6(216 - 1)\\5(6) + 5(36) + 5(216) + 5(1296) &= 6(1296 - 1)\end{aligned}$$

44.
$$3 = \frac{3(3-1)}{2}$$
$$3 + 9 = \frac{3(9-1)}{2}$$
$$3 + 9 + 27 = \frac{3(27-1)}{2}$$
$$3 + 9 + 27 + 81 = \frac{3(81-1)}{2}$$

45.
$$\frac{1}{2} = 1 - \frac{1}{2}$$
$$\frac{1}{2} + \frac{1}{4} = 1 - \frac{1}{4}$$
$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = 1 - \frac{1}{8}$$
$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = 1 - \frac{1}{16}$$

46.
$$\frac{1}{1 \cdot 2} = \frac{1}{2}$$
$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} = \frac{2}{3}$$
$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} = \frac{3}{4}$$
$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} = \frac{4}{5}$$

Legend has it that the great mathematician Carl Friedrich Gauss (1777–1855) at a very young age was told by his teacher to find the sum of the first 100 *counting numbers. While his classmates toiled at the problem, Carl simply wrote down a single number and handed the correct answer in to his teacher. The young Carl explained that he observed that there were* 50 *pairs of numbers that each added up to* 101. *(See below.) So the sum of all the numbers must be* $50 \times 101 = 5050$.

$$1 + 2 + 3 + \cdots + 98 + 99 + 100$$

101

101

101

50 sums of 101 = 50 × 101 = 5050

Use the method of Gauss to find each sum.

47. $1 + 2 + 3 + \cdots + 200$

48. $1 + 2 + 3 + \cdots + 400$

49. $1 + 2 + 3 + \cdots + 800$

50. $1 + 2 + 3 + \cdots + 2000$

51. Modify the procedure of Gauss to find the sum $1 + 2 + 3 + \cdots + 175$.

52. Explain in your own words how the procedure of Gauss can be modified to find the sum $1 + 2 + 3 + \cdots + n$, where n is an odd natural number. (When an odd natural number is divided by 2, it leaves a remainder of 1.)

53. Modify the procedure of Gauss to find the sum $2 + 4 + 6 + \cdots + 100$.

54. Use the result of **Exercise 53** to find the sum $4 + 8 + 12 + \cdots + 200$.

55. What is the most probable next number in this list?

12, 1, 1, 1, 2, 1, 3

(*Hint:* Think about a clock with chimes.)

56. What is the next term in this list?

O, T, T, F, F, S, S, E, N, T

(*Hint:* Think about words and their relationship to numbers.)

57. Choose any three-digit number with all different digits, and follow these steps.

(a) Reverse the digits, and subtract the smaller from the larger. Record your result.

(b) Choose another three-digit number and repeat this process. Do this as many times as it takes for you to see a pattern in the different results you obtain. (*Hint:* What is the middle digit? What is the sum of the first and third digits?)

(c) Write an explanation of this pattern.

58. Choose any number, and follow these steps.

(a) Multiply by 2.

(b) Add 6.

(c) Divide by 2.

(d) Subtract the number you started with.

(e) Record your result.

Repeat the process, except in Step (b), add 8. Record your final result. Repeat the process once more, except in Step (b), add 10. Record your final result.

(f) Observe what you have done. Then use inductive reasoning to explain how to predict the final result.

59. Complete the following.

$142{,}857 \times 1 =$ ______	$142{,}857 \times 2 =$ ______
$142{,}857 \times 3 =$ ______	$142{,}857 \times 4 =$ ______
$142{,}857 \times 5 =$ ______	$142{,}857 \times 6 =$ ______

What pattern exists in the successive answers? Now multiply 142,857 by 7 to obtain an interesting result.

60. Refer to **Figures 2(b)–(e)** and **Figure 3.** Instead of counting interior regions of the circle, count the chords formed. Use inductive reasoning to predict the number of chords that would be formed if seven points were used.

1.2 AN APPLICATION OF INDUCTIVE REASONING: NUMBER PATTERNS

OBJECTIVES

1. Be able to recognize arithmetic and geometric sequences.
2. Be able to apply the method of successive differences to predict the next term in a sequence.
3. Be able to recognize number patterns.
4. Be able to use sum formulas.
5. Be able to recognize triangular, square, and pentagonal numbers.

Number Sequences

An ordered list of numbers such as

$$3, 9, 15, 21, 27, \ldots$$

is called a *sequence*. A **number sequence** is a list of numbers having a first number, a second number, a third number, and so on, called the **terms** of the sequence.

The sequence that begins

$$5, 9, 13, 17, 21, \ldots$$

is an *arithmetic sequence,* or *arithmetic progression*. In an **arithmetic sequence,** each term after the first is obtained by adding the same number, called the **common difference,** to the preceding term. To find the common difference, choose any term after the first and subtract from it the preceding term. If we choose $9 - 5$ (the second term minus the first term), for example, we see that the common difference is 4. To find the term following 21, we add 4 to get $21 + 4 = 25$.

The sequence that begins

$$2, 4, 8, 16, 32, \ldots$$

is a *geometric sequence,* or *geometric progression*. In a **geometric sequence,** each term after the first is obtained by multiplying the preceding term by the same number, called the **common ratio.** To find the common ratio, choose any term after the first and divide it by the preceding term. If we choose $\frac{4}{2}$ (the second term divided by the first term), for example, we see that the common ratio is 2. To find the term following 32, we multiply by 2 to get $32 \cdot 2 = 64$.

EXAMPLE 1 Identifying Arithmetic and Geometric Sequences

For each sequence, determine if it is an *arithmetic sequence,* a *geometric sequence,* or *neither*. If it is either arithmetic or geometric, give the next term in the sequence.

(a) 5, 10, 15, 20, 25, . . . **(b)** 3, 12, 48, 192, 768, . . . **(c)** 1, 4, 9, 16, 25, . . .

Solution

(a) If we choose *any* term after the first term, and subtract the preceding term, we find that the common difference is 5.

$$10 - 5 = \mathbf{5} \qquad 15 - 10 = \mathbf{5} \qquad 20 - 15 = \mathbf{5} \qquad 25 - 20 = \mathbf{5}$$

Therefore, this is an arithmetic sequence. The next term in the sequence is

$$25 + \mathbf{5} = 30.$$

(b) If any term after the first is multiplied by 4, the following term is obtained.

$$\frac{12}{3} = \mathbf{4} \qquad \frac{48}{12} = \mathbf{4} \qquad \frac{192}{48} = \mathbf{4} \qquad \frac{768}{192} = \mathbf{4}$$

Therefore, this is a geometric sequence. The next term in the sequence is

$$768 \cdot \mathbf{4} = 3072.$$

(c) Although there is a pattern here (the terms are the squares of the first five counting numbers), there is neither a common difference nor a common ratio. This is neither an arithmetic nor a geometric sequence. ■